This version: April 2010

Entrepreneurship and the Hidden Economy: an Extended Matching Model [*]

Gaetano Lisi and *Maurizio Pugno*[♠]
(University of Cassino)

Abstract

This paper develops a labour market matching model to address the problem of the persistence of the hidden sector and of its regional concentration, as in Italy and in the enlarged Europe. The main novel features of the model are that entrepreneurial ability is heterogeneous, and that regular firms receive negative externalities from the hidden sector, which may capture the pressure typically exerted by corruption and organised crime, and positive externalities from the other regular firms. An interior equilibrium emerges; if externalities are non-linear, two equilibria are possible, thus accounting for regional dualism. The "bad" equilibrium is in fact characterised, with respect to the "good" one, by a larger hidden sector, lower levels of overall productivity, output, entrepreneurial ability used, extra-profits, wages, as well as positive externalities; while the negative externalities are relatively greater.

JEL classification: E26, J23, J24, J63, J64, L26

Keywords: entrepreneurship, hidden economy, shadow economy, underground economy, multiple equilibria, matching models

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[*] *DIPSE Working Paper*, 4/2009. Paper presented at the International Workshop on "*Entrepreneurship, Creativity and Innovation*", Cassino, April 2-3, 2009, and at the International Conference on "*Poverty Traps: an Empirical and Theoretical Assessment*", Naples, October 30-31, 2009. New version written for the International Workshop on "*Shadow Economy, Tax Policy, and the Labour Markets in an International Comparison: Options for Economic Policy*", University of Potsdam and DIW Berlin, Potsdam, 15-16, April 2010.

[♠] Corresponding author. Department of Economic Sciences and CreaM, University of Cassino, via S. Angelo, I-03043 Cassino (FR), Italy.
Tel.: +39 0776 2994702, fax +39 0776 2994834; e-mail: m.pugno@unicas.it.

1 – Introduction

This paper addresses the problem of the hidden sector as a persistent and backward component of the economy, and as relatively concentrated in specific regions. The typical example is the hidden sector in Italy, which consists of many small firms and productive activities framed in an advanced economic and institutional setting but also localised and linked to the specific socio-economic context of the country's southern regions (ISTAT, 2005, 2008; Daniele and Marani, 2008). Another example is the enlarged Europe, because the hidden sector is concentrated in the Eastern countries, which are also especially characterised by organised crime, corruption, and low law enforcement (Van Dijk, 2006; Johnson et al., 2000), but not necessarily by a heavy tax burden (Johnson et al., 1999).

A model is proposed to account for the persistency and localisation of the phenomenon of the hidden sector. It makes use of both fairly usual assumptions and key new ones. The former are the following: irregular firms adopt techniques which are relatively less efficient than those of official firms in order to produce the same product (Busato and Chiarini, 2004); they evade taxes despite the risk of being detected and punished (Boeri and Garibaldi, 2002, 2006); and they access informal production, thus avoiding greater start-up costs (Bouev, 2002, 2005). The new key assumptions are the following: firstly, entrepreneurial ability is heterogeneous across individuals; secondly, regular firms enjoy positive externalities from the other regular firms, but they also suffer from negative externalities produced by the hidden sector.

The paper adopts a matching model à la Pissarides (2000) extended to two productive sectors, to heterogeneous entrepreneurial ability, and to sectoral externalities. The extension of matching models to the hidden sector is not new in the literature (Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005; Kolm and Larsen, 2003; Fugazza and Jacques, 2004; Albrecht et al., 2009); but the other two extensions are novel, and they have interesting analytical consequences. For example, the zero-profit condition is usual in matching models for all firms because perfect competition prevails. But the assumption that entrepreneurial ability, which is a non-tradeable input for firms, is heterogeneous makes entry into the market not completely free. Therefore, the zero-profit condition holds only for the marginal individuals, i.e. those endowed with the minimum entrepreneurial ability; the other, abler, individuals become entrepreneurs because they earn extra-profits. The heterogeneity assumption will provide a new solution to the problem of determining a mixed allocation of vacant jobs between the regular and the irregular sector.

The extension of the model to sectoral externalities is based on the idea that the entrepreneurial ability is embedded in a socio-economic context which may be unfavourable because of high transaction costs. The entry to regular production may be hindered by various forms of rent stemming from corruption and criminal activity, but also by substituting market connections with family rent-seeking connections.[1] By contrast, regular firms may find a favourable socio-economic context when networking with the other firms is easy and trust prevails.

The paper is thus able to give theoretical account of a number of facts: at the macroeconomic level, the persistence of a substantial proportion of the hidden sector with detrimental effects on overall output and underemployment; at the microeconomic level, some key characteristics of irregular firms, such as their relatively lower entrepreneurial ability, lower profits and wages. Furthermore, the model is able to enlighten the intricate relationship between unemployment and hidden economy.

When the analysis concentrates on the role of externalities, the paper shows other results by recognising the particular non-linearity of externalities in diffusing themselves (Minniti, 2005; Ormerod, 2005; Puga and Venables, 1996; Krugman, 1991). Two macroeconomic equilibria may alternatively emerge within the same institutional structure and with the same economic potential. The "bad" equilibrium consists of a relatively large hidden sector, important negative externalities, and reduced positive externalities; the "good" equilibrium consists of a relatively small hidden sector, important positive externalities, and reduced negative externalities.

This approach to the problem of the hidden economy makes it possible to extend the opportunity of policy actions from the fine tuning of the institutional duties (Kolm and Larsen, 2003), from larger individual benefits of participating in the regular sector (Fugazza and Jacques, 2004), and from labour-market liberalisation (Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005), to actions intended to increase positive externalities and to reduce negative ones.

The paper is organised as follows: section 2 presents the benchmark model; section 3 extends the model to endogenous externalities; section 4 performs some numerical simulations; while section 5 concludes with some remarks on policy implications. The appendices set out the relevant proofs and math details.

[1] The southern regions of Italy are a typical case in which the socio-economic context of organized crime (Peri, 2004; Gpf-Ispo, 2005; Daniele and Marani, 2008), and of "amoral familism" (Banfield, 1958) has heavily burdened the economy.

2 – The benchmark model

The paper proposes a general equilibrium model of the matching type (Mortensen and Pissarides, 1994; Pissarides, 2000).[2] This means that both firm's equilibrium and aggregate equilibrium are studied, and that there are frictions in the labour market because firms and workers do not perfectly match. The general equilibrium character is particularly stressed, because the model considers two types of firms, thus forming two sectors, and because each firm is affected by the sectoral composition. The matching character of the model enables study of joint decision-making by entrepreneurs and workers, thus yielding non-market clearing wages in the two sectors, and unemployment.

The environment is characterised by a non-competitive labour market with wage bargaining.[3] Numerous firms competitively produce a homogeneous product, but adopt different institutional and technological set-ups.[4] They may be registered, and therefore pay a production tax and adopt a relatively advanced technology; or they may not be registered, and therefore evade taxes and adopt a less efficient technology. Hence non-registered firms form the hidden sector of the economy, which is illegal because of the process employed, not because of the good being produced.

An unexpected result of the model is that equilibrium is not necessarily a corner solution, as a competitive market for a homogeneous product might suggest. The key assumption for obtaining at least one interior solution is that entrepreneurs are endowed with different abilities. This is a new assumption in the family of matching models, and it allows us to concentrate on the conditions inducing entrepreneurs to enter one sector or the other.[5]

2.1 Entrepreneurs' expected profitability and workers' expected wages

As is usual in matching-type models (Pissarides, 2000; Petrongolo and Pissarides, 2001), let us assume that the meeting of vacant jobs and unemployed workers is regulated by an aggregate matching function with constant returns to scale. Let us denote the number of vacancies in the official (or regular) sector and in the hidden (or irregular) sector with v_r and

[2] Hence, as usually assumed, time is continuous, and individuals are risk neutral, live infinitely, and discount the future at an exogenous rate (*r*).

[3] In this work we abstract from goods and capital markets (both of which are assumed to clear) in order to highlight the joint effect of search frictions and rent sharing on job composition, rather than on prices of both capital input and final output.

[4] Indeed, «[...] *the underground economy model could be more appropriately defined as a "two technology model", since the same good is produced using two different technologies* » (Busato and Chiarini, 2004, p. 843).

[5] The literature that employs matching models instead concentrates on study of the individual's choice between running a firm or working as an employee (Fonseca et al., 2001; Pissarides, 2002; Uren, 2007).

v_s respectively, and the number of unemployed with u.[6] The matching function (*m*) in the two sectors is thus as follows (where *r* = *regular*, *s* = *shadow*):

$m_i = m(v_i, u)$ with $i \in \{r, s\}$

By assumption, the matching function is non-negative, increasing, concave and performs constant returns to scale, so that the job-finding rate, $g(\theta_i) = m(v_i, u)/u = m(\theta_i, 1)$, is positive, increasing and concave in "market tightness" defined as the ratio of vacancies to unemployment, $\theta_i = v_i / u$. Analogously, the rate at which vacancies are filled, $f(\theta_i) = m(v_i, u)/v_i = m(1, \theta_i^{-1})$, is a positive, decreasing and convex function of market tightness. Further, the *Inada*-type conditions hold: $\lim_{\theta_i \to 0} f(\theta_i) = \lim_{\theta_i \to \infty} g(\theta_i) = \infty$; $\lim_{\theta_i \to \infty} f(\theta_i) = \lim_{\theta_i \to 0} g(\theta_i) = 0$, with $i \in \{r, s\}$.

The Bellman equations specified to find infinite horizon steady-state solutions are:

Value of …	***Hidden sector***	***Official sector***
a vacancy	$r \cdot V_s = -c_s + f(\theta_s) \cdot [J_s - V_s]$	$r \cdot V_r = -c_r + f(\theta_r) \cdot [J_r - V_r]$
a filled job	$rJ_s = x_s y_s - w_s - \rho\phi\tau + (\delta + \rho) \cdot [V_s - J_s]$	$rJ_r = (px_r y_r + k) - w_r - \tau - s + \delta[V_r - J_r]$
searching a job	$r \cdot U_s = g(\theta_s) \cdot [W_s - U_s]$	$r \cdot U_r = g(\theta_r) \cdot [W_r - U_r]$
being employed	$r \cdot W_s = w_s + (\delta + \rho) \cdot [U_s - W_s]$	$r \cdot W_r = w_r + \delta \cdot [U_r - W_r]$

where V_i is the value of a vacancy; J_i is the value of a filled job; U_i is the value for seeking a job[7]; W_i is the value for being employed; c_i is the start-up cost; $p > 1$ is the exogenous productivity premium in the official sector; x_i is the entrepreneurial ability; y_i is the labour productivity; w_i is the wage; τ is an exogenous production tax; ρ is the exogenous instantaneous probability of a firm being discovered as unregistered; ϕ is the exogenous multiplier of the tax due to be levied (i.e. $\phi\tau$ is the penalty); δ is the exogenous destruction rate. The symbols *k* and *s* denote the specific advantages and disadvantages for regular firms, like the benefits of participating in a larger information network and of receiving specific public services, and conversely, of paying bureaucratic and administrative costs, including

[6] The unemployed are the only job seekers in the labour market. In terms of flows, the model ignores on-the-job-search and direct transitions from shadow to legal employment without intervening unemployment spells.

[7] The unemployed cannot search for a job in both sectors at the same time (i.e. there is directed search). However, irrespective of the sector, if an unemployed fails to find a job, he/s falls back in the same pool of unemployment.

bribes and money protection if imposed by criminal organisations.[8] Since $p>1$ captures the greater technological level of the official sector, we can assume for simplicity that $y_r = y_s$.

As usual, wages are assumed to be the outcome of a Nash bargaining problem:

$$w_r = arg\,max\{(W_r - U_r)^{\beta} \cdot (J_r - V_r)^{1-\beta}\} \Rightarrow (W_r - U_r) = \frac{\beta}{(1-\beta)} \cdot (J_r - V_r)$$

$$w_s = arg\,max\{(W_s - U_s)^{\gamma} \cdot (J_s - V_s)^{1-\gamma}\} \Rightarrow (W_s - U_s) = \frac{\gamma}{(1-\gamma)} \cdot (J_s - V_s)$$

where $\beta \in (0,1)$ is the surplus share for labour in the official sector. Analogously, the wage rate in the irregular firm is obtained with a share $\gamma \in (0,1)$. Simple manipulations thus yield the formulae for wages:

$$w_r = (1-\beta) \cdot rU_r(\theta_r) + \beta \cdot (px_r y_r + k - s - \tau - rV_r(\theta_r))$$
$$w_s = (1-\gamma) \cdot rU_s(\theta_s) + \gamma \cdot (x_s y_s - \rho\phi\tau - rV_s(\theta_s))$$

with $w_i'(\theta_i) > 0 \;\; \forall i$, since $V_i'(\theta_i) < 0$, and $U_i'(\theta_i) > 0 \;\; \forall i$.

The surplus of a job in each sector (divided between entrepreneur and worker by the wage) is defined as the sum of the worker's and firm value of being on the job, net of the respective outside options, so that:

$$S_i = J_i - V_i + W_i - U_i \qquad \text{with } i \in \{r,s\}$$

making use of the Bellman equations, we get:

$$S_s = \frac{x_s \cdot y_s - \rho\phi\tau + c_s}{r + \delta + \rho + (1-\gamma) \cdot f(\theta_s) + \gamma \cdot g(\theta_s)}; \qquad S_r = \frac{p \cdot x_r \cdot y_r + k - s - \tau + c_r}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)}.$$

Note that both the surplus and wages are heterogeneous within the sectors, besides different between the two sectors. This is because of the overall heterogeneity of entrepreneurial ability. Since $(J_s - V_s) = (1-\gamma) \cdot S_s$ and $(J_r - V_r) = (1-\beta) \cdot S_r$, it is straightforward to get:

$$rV_s(x) = \frac{f(\theta_s) \cdot (1-\gamma) \cdot (x_s \cdot y_s - \rho\phi\tau) - c_s \cdot (r + \delta + \rho + \gamma \cdot g(\theta_s))}{r + \delta + \rho + (1-\gamma) \cdot f(\theta_s) + \gamma \cdot g(\theta_s)} \qquad [1]$$

$$rV_r(x) = \frac{f(\theta_r) \cdot (1-\beta) \cdot (p \cdot x_r \cdot y_r + k - s - \tau) - c_r \cdot (r + \delta + \beta \cdot g(\theta_r))}{r + \delta + (1-\beta) \cdot f(\theta_r) + \beta \cdot g(\theta_r)} \qquad [2]$$

As in Fonseca et al. (2001), we ignore the range beyond which θ_i is large enough to turn rV_i negative. Hence, it must be that $\theta_i \in [0, \tilde{\theta}_i) \;\; \forall i$, where $\tilde{\theta}_i < \infty$ is the value such that $V_i(\tilde{\theta}_i) = 0$. Furthermore, since for $\theta_i = 0$ the vacancy would be always filled, the relevant interval for θ_i becomes $\theta_i \in (0, \tilde{\theta}_i) \;\; \forall i$.

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[8] Both s and k are assumed as parameters in this section, but they will be considered as variables in section 3.

2.2 Entrepreneurial ability and the career choice

A key feature of the model is that the comparison of expected profitability of posting vacancies in the two sectors depends on the entrepreneurial ability of individuals (x). Each individual is in fact assumed to be endowed with a specific entrepreneurial ability, and all individuals are heterogeneous with respect this ability. Formally, entrepreneurial ability x is distributed over a continuum of infinitely-living individuals who expect to enter the labour market, and it can be measured in continuous manner, $x \in [0, x_{max}]$, following the known cumulative distribution function $F(x)$ on the support of $[0,1]$.

The minimum ability required to open no vacancy in the hidden sector can be obtained in a very simple way from the *free-entry* condition, i.e. from $V_s = 0$ in equation [1]:

$$\lim_{v_s \to 0}\left[\frac{c_s}{f(\theta_s)} = \frac{(1-\gamma)\cdot(x\cdot y_s - \rho\phi\tau)}{(r+\delta+\rho+g(\theta_s))}\right] \Rightarrow x_{min} = \frac{\rho\phi\tau}{y_s} > 0$$

hence, when a positive θ_s is determined (see below), then the individuals endowed with $x > x_{min}$ become entrepreneurs because they will earn positive profits in posting vacancies. The minimum level of ability $x_{\min}$ is the threshold for individuals to become entrepreneurs in the hidden sector, but it is also the threshold to become entrepreneurs generally, because the level of ability required to enter the regular sector is even higher, as will shortly be made clear. Since ability is not tradeable, all the individuals endowed with $x > x_{min}$ will earn extra-profit as a rent in posting vacancies, i.e. $V_i > 0$, with $i \in \{r,s\}$. Accordingly, for an equal or smaller level of ability, individuals become workers (measured by l) and then they do not post any vacancy.

Let us then define a threshold level of entrepreneurial ability $T \in \,]x_{min}, x_{max}]$ such that two entrepreneurs drawn from the two sectors yield equal expected profitability, i.e.:

$$V_r(x=T) = V_s(x=T) \qquad [3]$$

Therefore, T can be derived in a straightforward way from equations [1], [2], and [3]:

$$T = \frac{\dfrac{(\tau+s-k)+c_r\cdot A}{A+1} - \dfrac{\rho\phi\tau + c_s\cdot B}{B+1}}{\dfrac{py_r}{A+1} - \dfrac{y_s}{B+1}} \qquad [4]$$

with $A \equiv \dfrac{r+\delta+\beta\cdot g(\theta_r)}{(1-\beta)\cdot f(\theta_r)}$ and $B \equiv \dfrac{r+\delta+\rho+\gamma\cdot g(\theta_s)}{(1-\gamma)\cdot f(\theta_s)}$.

In order to have a positive expression on the *r.h.s* of [4], the following restrictions are sufficient: $(\tau+s-k) > \rho\phi\tau$, $(\tau+s-k) > c_s$, $c_r > \rho\phi\tau$, and p must be sufficiently great (see

Appendix A for the details). The first three restrictions are realistic,[9] as they also emerge from calibrations run in the literature (see section 4), the forth restriction is necessary for allowing the regular sector to survive. From these restrictions an interesting result can be obtained, having observed that the intercept of $V_r(x)$ emerges as more negative than the intercept of $V_s(x)$, and the slope of $V_r(x)$ emerges as steeper than the slope of $V_s(x)$.

Remark 1. *Official jobs are ran by the relatively abler entrepreneurs (see figure 1).*

========== *Fig. 1 about here (now at the end)* ==========

This is one of the key results of the benchmark model: entrepreneurs will prefer the hidden sector if they are endowed with $x < T$, and they will prefer the official sector if they are endowed with $x > T$. Some entrepreneurial ability may thus remain hidden, but it will also be of the worst quality (see also Pugno, 2000a; Carillo and Pugno, 2004; Rauch, 1991, Levenson and Maloney, 1998). This result runs counter to the argument that the shadow sector is an incubator of infant industries: in fact, regular firms are more productive because they are run by more able entrepreneurs.

From the macroeconomic point of view, the entrepreneur's indifference condition [3] implies that the share of entrepreneurs who opens a vacancy in the hidden sector is $F(T)-l=v_s$, while the share $1-F(T)=v_r$, opens a vacancy in the official sector. Entrepreneurs may thus post a vacancy and then fill the job, or fail to fill it, in one of the two sectors, so that it can be simply stated that $v_r = 1-(v_s+l)$.[10] Hence, equation [4] can be re-written in a more general form as follows:

$$T = T(v_s) \qquad [4']$$

since u is given to the entrepreneurs. The property that $\partial T/\partial v_s < 0$ follows from the restrictions on the parameters in [4] (see *Appendix A*). Equations $T = T(v_s)$ can be coupled with the equation $v_s = v_s(T)$, which depends on the distribution of ability across entrepreneurs, and it is monotonically rising in T, from x_{min} up to x_{max}. Both equations can be

[9] For example, the value of the start-up cost in the hidden sector c_s should be very low, since the ease of entry is often used as one of the criterion for defining the informal sector (Gërxhani, 2004). On the contrary, the start-up cost c_r is often very heavy. This can be explained by higher entrance barriers into the official sector or access costs to legality associated with excessive regulations, administrative burdens, licence fees, bribery (Bouev, 2005).

[10] In order to focus the attention on the entrant entrepreneurs, in this model the number of incumbent entrepreneurs is exogenous and outside of the population. Hence, $v_r + v_s + u + 2(n_r + n_s) = 1 + n_r + n_s$. Note that the relevant interval $\theta_i \in (0, \tilde{\theta}_i)$ implies that $u \neq 0$ and $v_i \neq 0 \ \forall i$.

represented in the diagram with axes [v_s, T] like fig. 2. Equation [4'] has been built for $T \in [\, x > x_{min}, x_{max}\,]$, so that its vertical start-point is higher than the intercept of $v_s = v_s(T)$.

========== *Fig. 2 about here (now at the end)* ==========

Remark 2. *A unique couple of (v_s , T) exists in the model.*

Note that this key result rules out the possibility of a perverse equilibrium which implies that the abler entrepreneurs enter the shadow sector.

2.3 The unemployment equation

Although the economy has two sectors, empirically we observe a unique rate of unemployment. Unemployment is the difference between the labour force and the sum of workers employed in official and underground sectors by definition. Since the total share of workers in the population is *l*, the unemployment identity requires:

$$u_r + u_s \equiv u = l - n_r - n_s \qquad [5]$$

where n_r and n_s are the steady-state employment in the official and hidden sectors, respectively. Since jobs arrive to unemployed workers at rate $g(\theta_i)$, with $i \in \{r, s\}$, and regular and irregular filled jobs are destroyed at rate δ and $(\delta + \rho)$, respectively, the equations for the evolution of employment in the two sectors in terms of the workers transition rates are the following:

$$\dot{n}_r = u \cdot g(\theta_r) - \delta \cdot n_r$$

$$\dot{n}_s = u \cdot g(\theta_s) - (\delta + \rho) \cdot n_s$$

In steady-state ($\dot{n}_r = \dot{n}_s = 0$), we get:

$$n_r = \frac{u \cdot g(\theta_r)}{\delta} \qquad [6]$$

$$n_s = \frac{u \cdot g(\theta_s)}{\delta + \rho} \qquad [7]$$

Steady-state unemployment is thus given by [5], [6] and [7]:

$$u = \frac{l}{\dfrac{g(\theta_r)}{\delta} + \dfrac{g(\theta_s)}{\delta + \rho} + 1} \qquad [8]$$

Equation [8] closes the model, since *u*, which has been given to the entrepreneurs, can thus be determined, and the following result can be drawn:

Proposition. *An aggregate equilibrium with positive u exists and it is unique. The qualitative results obtained in partial equilibrium, where u is given, also hold in general equilibrium, where u is endogenous (see Appendix B for proofs).*

Hence, the equilibrium of the model can be defined thus:

Definition. *The solutions for the four key variables* v_s, v_r, T *and* u *are obtained by considering: 1) the Bellman equations; 2) the entrepreneur's indifference condition between running firms in the two sectors, given their entrepreneurial ability distribution; 3) the unemployment identity and the equilibrium condition of the transition flows on the supply side of the labour market.*

2.4 Discussion

The main result is that an interior solution exists where both the hidden sector and the official sector survive in equilibrium (see also Pugno, 2000a, and Carillo and Pugno, 2004). This may explain the so-called "shadow puzzle", i.e. the persistence of the hidden sector despite advances in detection technologies and organisation by public authorities to reduce irregularities (Boeri and Garibaldi, 2006).

A number of other important results can be drawn from exercises of comparative statics. A general exercise concerns the effects of the shift of the *T*-curve [4] due to changes in some parameters. Its downward shift decreases both the (partial) equilibrium of v_s in fig. 2, and the (general) equilibrium of v_s of the model. Therefore, the downward shift of the *T*-curve [4] squeezes the proportion of the hidden sector and expands the proportion of the official sector, as clearly emerges from the definitions of v_s and v_r, and as it can be easily derived by equations [5], [6] and [7] jointly.

The downward shift of the *T*-curve [4] can thus increase overall output, because it increases the proportion of the most productive sector. The official sector is in fact more productive than the hidden sector because the official sector exhibits the premium *p*, which captures its greater technological level, and the most able entrepreneurs prefer this sector.

The downward shift of the *T*-curve [4] also increases the shadow wage gap, i.e. the wage differentials between the two sectors. This effect is due to the rise of the equilibrium level of v_r, since the wages are increasing functions with respect to the vacancies level.

2.4.1 Underground economy, unemployment and the efficient monitoring *

The net result of vacancies composition on unemployment depends on the steepness of the Beveridge Curves. Indeed, the steeper is the (negative) vacancy-unemployment relationship, the stronger is the effect due to a vacancies change.

From equation [8], the steepness of the Beveridge Curves depends on the probability to find a job in each sector, the job destruction rate and the monitoring rate. The probability to find a job depends on the properties of the matching function, the job destruction rate depends on the business cycle (precisely, it increases in slump), while the monitoring rate is a policy parameter. Hence, there is a scope for government policy and the monitoring rate play a key role.

Precisely, in extreme case in which the monitoring is null ($\rho = 0$), the *Beveridge Curve* of the irregular sector is steeper than the *Beveridge Curve* of the official sector (see *Appendix C*). Hence, this means that when the hidden vacancies decrease, then the unemployment rate increases. This conclusion runs counter to Bouev's (2002, 2005) idea that scaling down the unofficial sector can lead to a decrease in the level of unemployment, whereas it agrees with the idea of Boeri and Garibaldi (2002, 2006) that attempts to reduce, in the first place, shadow employment will result in higher open unemployment.

However, a positive level of monitoring ($\rho > 0$) is a necessary condition to preserve legal jobs and it is possible to show that for a sufficient level of monitoring is the *Beveridge Curve* of the official sector to be steeper (see again *Appendix C*). Hence, this means that when the irregular vacancies decrease and the official vacancies increase, then the unemployment rate decreases. In this case, as obtained by simulations in Boeri and Garibaldi (2006), unemployment and underground employment "… *are two face of the same coin* ". As a result, an adequate level of monitoring, i.e. $\rho > \{\delta \cdot [g'(\theta_s) / g'(\theta_r) - 1]\} \equiv \sigma$, can reduce both underground economy (reducing the value of a filled irregular job) and unemployment (making steeper the *Beveridge Curve* of the official sector). Furthermore, since $\partial\rho / \partial\delta > 0$, the efficient monitoring is higher in slump, when the "shadow-option" becomes more attractive.[11]

In short, if the monitoring rate ρ is set in an efficient way, i.e. it is higher than the threshold value of efficiency σ, any policy directed to reduce the irregular sector also reduces

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* *Preliminary version*.

[11] Usually, the policy maker is unwilling to do more inspections in slump. However, policies such as tax reduction and tax amnesty does not seem to achieve great results. Indeed, the lack of an efficient monitoring could explain the persistence of the shadow economy also in the developed economies.

the unemployment rate.[12] Otherwise, open unemployment should be substitutable with employment in the hidden sector.

3 – The model with endogenous externalities

The performances of the regular firm and the irregular firm differ not only because of their technological level and other specific economic features but also because of the contexts in which they operate.

If regular firms are diffused and pervasive in the economy with respect to the irregular firms, they operate more efficiently than in the case where they are relatively few. In fact, information flow more easily, trust is more widespread, networking is more diffused, and a more efficient use of public services, including information and assistance from the public authorities and agencies, becomes possible. Large positive externalities are at work in this case.[13]

By contrast, if the hidden sector is widespread, large negative externalities on the regular firms may be at work. The unfortunate case of the southern regions of Italy provides the clearest example of these externalities, because in those regions the hidden sector is linked to the illegal sector and to criminal organisations. Transaction costs become greater in this case, market networking becomes distorted, and tax morality worsens.[14]

Both positive and negative externalities can be characterised by a non-linearity which is typical of diffusion of the contagion-type. In the case of positive externalities, the diffusion of information and of trustful entrepreneurial behaviour typically follows the bandwagon effect, which characterises the acceleration of the central phase of the diffusion process (Minniti, 2005). A similar pattern seems to be exhibited by criminal behaviour (Glaeser et al., 1996) and criminal enterprises (Pugno, 2000b), which exert negative externalities on regular firms. The *S-shape* pattern of diffusion is based on Schelling's argument (1978: ch.3) of critical mass in imitative behaviour on the spatial dimension (see also Granovetter 1978). The

[12] Hence, open unemployment and employment in the hidden sector may have changes of the same sign.

[13] There is a large body of evidence for the spillover effects on productivity. See Cooper and Haltiwanger (1996) for a survey on this literature. For the importance of social networks for entrepreneurship see Aldrich and Zimmer (1986), and Granovetter (1985).

[14] Cross-section analysis of developed and developing countries shows that the size of the hidden sector is significantly negatively correlated with generalised trust (D'Hernoncourt and Méon, 2008), and that generalised trust is negatively correlated with corruption. Although the connection between trust and corruption is reciprocal, the effect of trust on corruption is greater than the reverse (Uslaner, 2002). Further, hidden activity is larger in countries where managers are more likely to pay bribes, where managers pay for mafia-type protection, where managers have less faith in the legal system (Johnson et al., 2000), and where corruption is generally more widespread (Buehn and Schneider, 2009).

non-linear diffusion also emerges if imitation simply follows costs reduction because of strategic complementarities on the spatial dimension, thus explaining geographical concentration (Krugman, 1991; Puga and Venables, 1996).

Our model is able to capture these phenomena with interesting results. Let us cease considering s and k as fixed parameters and treat them as functions of v_r and v_s as follows:

$$s = \frac{\Phi_1}{1 + e^{\Phi_2 - \Phi_3 v_s}} \qquad [9]$$

$$k = \frac{\Omega_1}{1 + e^{\Omega_2 - \Omega_3 v_r}} \qquad [10]$$

The key property of [9], which is monotonically increasing with respect to v_s, is the convexity in the first trait and then the concavity. Function [10] has the same properties with respect to v_r, but opposite properties with respect to v_s, so that the algebraic sum $s - k$ reinforces the non-linear effect in the same direction. Greek capital letters denote the horizontal position of the inflection point, if numbered with 2, and the slope of the function, if numbered with 3. The adoption of these specifications fixes the ideas without losing in generality. The parameter Φ_1 captures the administrative and bureaucratic burdens and the maximum burden imposed by the criminal context, while Φ_3 denotes the acceleration effect when the critical density of the criminal activity has been approached. Similarly, Ω_1 captures the maximum possible effect of the positive externalities arising from the diffusion of regular firms, while Ω_3 denotes the acceleration effect of these externalities.

If the functions $s(v_s)$ and $k(1 - l - v_s)$ as in [9] and [10] are plugged into [4], then the relationship between T and v_s can change significantly, because a "hump" arises in the representation on the (v_s, T)–axes. For v_s close to zero, negative externalities tend to the minimum, and positive externalities tend to the maximum. For rising v_s, the threshold value of entrepreneurial ability T is declining when v_s remains low, but it rises when the density of the irregular firms accelerates the negative externalities and largely reduces the positive externalities, since greater entrepreneurial ability is required. After this acceleration of the externalities, the usual forces that reduce function [4] once again prevail, thus going towards the conditions where negative externalities are at the maximum, and positive externalities are at the minimum, since v_s becomes predominant. This captures two distinct facts: that a widespread hidden sector discourages the establishment of regular firms, thus reducing the proportion of the official sector; and that efficient networking requires numerous official partner firms.

If the accelerations and decelerations are irrelevant and externalities diffuse themselves smoothly, then crossing the extended variant of [4] with $v_s(T)$ determines the unique equilibrium in a way similar to that in the benchmark case; but the difference is that the proportion of the hidden sector is greater if negative externalities are greater than the positive ones. However, if accelerations and decelerations are significant and externalities diffuse themselves roughly, then three intersections become possible, as depicted in fig. 2 (dotted line).[15] The two extreme equilibria may be labelled as "good" and "bad" because they define two different conditions where the proportion of the hidden sector is small and, respectively, large; production is high and, respectively, low; the entrepreneurial ability is used efficiently and, respectively, inefficiently; shadow wage gap is high, and, respectively, low; negative externalities are limited, and, respectively, widespread; positive externalities are exploited, and, respectively, scarce. Furthermore, since several workers in the irregular firms appear as unemployed in the official statistics, the so-called disguised unemployment is higher in the "bad" with respect to the "good" equilibrium.

This result is interesting because it can represent an economy characterised by a uniform structure, including the institutional structure, as captured by the same parameters of the model, but with two regions and two populations that differ in their histories alone, as captured by different initial levels of v_s. The region starting with a greater proportion of the hidden sector may converge towards the "bad" equilibrium, while the region starting with a smaller proportion of the hidden sector may converge towards the "good" equilibrium. Distortions, both costly and beneficial, develop differently, and eventually establish a dualism in both economic and social aspects. The Italian North-South divide, which is special but not unique in the world, can thus find an explanation.

Therefore, these results strengthen the idea of the persistence of shadow activities, since they emerge as an avoidable phenomena in so far as the hidden sector is really small in the "good" equilibrium. Symmetrically, these results weaken the idea that shadow activities are incubators of infant industries because they attract the less qualified entrepreneurs and workers even in the case of a substantial hidden sector.

[15] Also Minniti's (2005) model of entrepreneurship and non-linear externalities, but without the hidden sector, exhibits multiple equilibria.

4 – Simulations and comparative statics

In order to substantiate the main analytical predictions of the theoretical model, some simple numerical simulations are now performed. The baseline specification of the model's parameters has been drawn from Boeri and Garibaldi (2006), and it is described in Table 1.[16]

Table 1. Model's calibration (source: Boeri and Garibaldi, 2006)

parameter	*notation*	*calibration value*
workers' surplus share (regular sector)	β	0.50
discount rate	r	0.03
monitoring rate	ρ	0.06
destruction rate	δ	0.15
production tax	τ	0.20
unemployment rate (sectors average)	u	0.0981
search cost	c	0.40
matching elasticity (Cobb-Douglas)	a	0.50

Further, according to Italian tax law, ϕ is calibrated equal to 1.3 (see Busato and Chiarini (2004), who use similar values for τ and ρ).

This paper follows the bulk of the existing literature and assumes a Cobb-Douglas matching function (Petrongolo and Pissarides, 2001; Stevens, 2004):

$m = v_i^{1-a} \cdot u^a$ with $i \in \{r, s\}$

The simulation confirms that the function $T(v_s)$ is monotonically decreasing in v_s, and the vertical starting-point of $T(v_s)$ is clearly higher than the intercept of $v_s(T)$. Regarding function $v_s(T)$, we use a distribution for the entrepreneurial ability x that is negative exponential.[17] As a result, a unique couple of (v_s, T) exists in the benchmark model.

========== *Fig. 3 about here (now at the end)* ==========

The effects of parameters change, which are interesting for policy purposes, on the interior equilibrium are summarized in Table 2 below:

Table 2. The effects of policy parameters change

Policy \ *Effect on*	T	v_s	v_r	θ_r / θ_s
$\Delta\rho > 0$	_	_	+	+
$\Delta\phi > 0$	_	_	+	+

[16] The productivity premium is calibrated so as to ensure T > 0 (see *Appendix A*).

[17] A negative exponential distribution is used by Boeri and Garibaldi (2006) for the distribution of productivity.

$\Delta\tau > 0$	+	+	–	–
$\Delta c_r > 0$	+	+	–	–

The effects of a change in the intensity of monitoring ($\Delta\rho > 0$), the severity of punishment for concealing business underground ($\Delta\phi > 0$), as well as the effect of higher taxes ($\Delta\tau > 0$), confirm the results obtained in other studies on shadow economies (Friedman et al., 2000; Johnson et al., 2000; Sarte, 2000). In our model, c_r reflects the *barriers on entrepreneurship* in the official sector: a shadow employment that is increasing in labour market regulations is common to other models of the shadow economy and holds in many cross-sectional studies (Bouev, 2005).

Finally, regarding the extended model, the calibration considers the parameter values in equations [9] and [10] so as to ensure that $T > 0$. The result is also depicted in fig. 3. In particular, the simulation shows the special role played by the parameters which regulate the acceleration/deceleration of the externalities: in fact, the greater is Φ_3 and the lower is Ω_3, the higher is the "hump" of the extended function [4], because the negative externalities rise faster and the positive ones end up quicker.

5 – Final remarks

This paper has proposed a model able to account for the persistence and the localisation of the hidden sector. The persistence is captured by the interior equilibrium, where the hidden sector coexists with the regular sector. The key assumption yielding this result is a new one, i.e. the heterogeneous ability of entrepreneurs. The localisation of the hidden sector due to the socio-economic context is captured by the possibility of two equilibria, given the same structure of parameters, where the hidden sector may be substantial and negligible respectively, depending on the starting conditions, i.e. on history. The key assumption yielding this result is again a new one, i.e. sufficient negative externalities from the hidden sector, and positive externalities from the regular sector on regular firms.

The model also suggests some policy measures besides the more usual ones, although it is not designed to determine the optimal policy. Any policy action that discourages the profitability of irregular firms will very likely improve the overall production level and productivity, through the composition effect. Entrepreneurs take advantage of their abilities to "go over-ground", while tax morality is strengthened. But the extended model yields a further result, since it suggests policy actions from the sectoral perspective, rather than from the firm perspective alone, with possible powerful effects.

Policy measures may be directed at changing the externalities. In the case of negative externalities, the contagion effect should be combatted, for example, by supporting those firms which pledge not to pay bribes and protection money, and by building a virtuous network of customer, creditors, etc. for them. In the case of positive externalities, infrastructure, network facilities and specific public services for regular firms should be provided. These policy measures may be especially effective in that they can trigger an endogenous change from the equilibrium where the hidden sector is substantial to the equilibrium where the hidden sector is negligible.

Appendices

Appendix A: Properties of equation [4]

The threshold *T* is a special *x*, so that it must be positive since $x > x_{min} \geq 0$. Hence, also the *r.h.s* of [4] must be positive. The conditions for the positivity of the *r.h.s* of [4] are:

$$\frac{py_r}{A+1} > \frac{y_s}{B+1} \qquad \text{[A.1]}$$

$$\frac{(\tau+s-k)+c_r\cdot A}{A+1} > \frac{\rho\phi\tau+c_s}{B+1} \qquad \text{[A.2]}$$

Let us examine the limit of the previous key conditions for v_r (and v_s) which goes to zero.

- If $v_r \to 0$, then $A \to 0$ and $B \to \{0 < \overline{B} < \infty\}$, so that:

  $py_r > \frac{y_s}{\overline{B}+1}$, which is always true, since $y_r = y_s$, and p > 1, and

  $\tau+s-k > \frac{\rho\phi\tau+c_s\cdot\overline{B}}{\overline{B}+1} \Rightarrow \overline{B} > \frac{\rho\phi\tau-(\tau+s-k)}{\tau+s-k-c_s}$, which requires as sufficient conditions that $(\tau+s-k) > \rho\phi\tau$, $(\tau+s-k) > c_s$.

- If $v_s \to 0$, then $B \to 0$ and $A \to \{0 < \overline{A} < \infty\}$, so that:

  $\frac{py_r}{\overline{A}+1} > y_s \Rightarrow p > \frac{y_s\cdot(\overline{A}+1)}{y_r}$ which requires that *p* is sufficiently greater than 1,

  $\frac{\tau+s-k+c_r\cdot\overline{A}}{\overline{A}+1} > \rho\phi\tau \Rightarrow \overline{A} > \frac{\rho\phi\tau-(\tau+s-k)}{c_r-\rho\phi\tau}$, with $c_r > \rho\phi\tau$ as a sufficient condition to hold.

The proof that $\partial T/\partial v_s < 0$ in [4] thus becomes straightforward, having reminded that $1-l = v_s + v_r$, and that $\theta_i = v_i / u$. Since $\frac{\partial A}{\partial v_s} < 0$ and $\frac{\partial B}{\partial v_s} > 0$, the denominator of [4] is rising in v_s, i.e. $\frac{\partial}{\partial v_s}\left(\frac{py_r}{A+1}-\frac{y_s}{B+1}\right) > 0$, while, the numerator of [4] is decreasing in v_s:

$$\frac{\partial}{\partial A}\left(\frac{\tau+s-k+c_r\cdot A}{A+1}\right) = \frac{c_r-(\tau+s-k)}{(A+1)^2} > 0 \qquad \text{if } c_r > \tau+s-k$$

$$\frac{\partial}{\partial B}\left(\frac{\rho\phi\tau+c_s\cdot B}{B+1}\right) = \frac{c_s-\rho\phi\tau}{(B+1)^2} > 0 \qquad \text{if } c_s > \rho\phi\tau.$$

The complete restriction set of the parameters is thus: $c_r > (\tau+s-k) > c_s > \rho\phi\tau$. Note that these are sufficient but not necessary conditions to obtain $\partial T/\partial v_s < 0$.

Appendix B: Proof of the proposition in section 2.3

In order to prove the existence and uniqueness of the solution for *u*, let us rewrite equation [8] as follows:

$$u=\frac{l}{\frac{g(v_r/u)}{\delta}+\frac{g(v_s/u)}{\delta+\rho}+1}=\Gamma(u) \qquad [8']$$

It can be observed that u ranges between 0 and l (where $l < 1$), and that the *r.h.s.* of [8'] is a rising and concave function in u for given v_r and v_s, because $\partial g(\theta_i)/\partial u<0$ and $\partial^2 g(\theta_i)/\partial u^2>0$. Since $\lim_{u\to 0}\Gamma(u)=0$ and $\lim_{u\to l}\Gamma(u)<l$, because of the Inada conditions, a unique intersection exists between the *l.h.s.* and the *r.h.s.* of [8'].

Indeed, equation [8'] should be also rewritten as follows:

$$u=\frac{l}{\frac{g(v_r(u)/u)}{\delta}+\frac{g(v_s(u)/u)}{\delta+\rho}+1} \qquad [8'']$$

Note that induced changes of $v_r(u)$ and $v_s(u)$, through changes in T, cannot cumulate because v_r and v_s are complementary, being $1-l=v_r+v_s$. Further, since the properties of $\Gamma(u)$ hold even if either v_r or v_s goes to zero, a unique intersection also exists between the *l.h.s.* and the *r.h.s.* of [8''].

In order to prove that the qualitative results which are obtained in partial equilibrium also hold in general equilibrium, it is sufficient to prove that:

$\text{sign}\left(\frac{\partial T(v_s)}{\partial v_s}\right)=\text{sign}\left(\frac{\partial T(v_s,u(v_s))}{\partial v_s}\right)<0$, where $u = u(v_s)$ is the explicit general form of [8']. This inequality follows from the conditions:

$\left(\frac{\partial A}{\partial u}\right)<0$, $\left(\frac{\partial B}{\partial u}\right)<0$, and $\left(\frac{\partial T}{\partial A}\right)>0$, $\left(\frac{\partial T}{\partial B}\right)<0$, as obtained in the *Appendix A* under the stated restrictions on the parameters. Hence, the effect of u on T is ambiguous but small, because the effects of the two sectors (by A and B) partially offset each other. Therefore, the net result is that induced changes of u through changes in some parameters cannot modify the qualitative result obtained in partial equilibrium.

Appendix C: Beveridge Curves analysis

From equation [8], it is straightforward to get the Beveridge Curve of both sectors:[18]

$$\frac{\partial u}{\partial v_r}=-\left\{\frac{l\cdot\delta\cdot(\delta+\rho)^2\cdot \overset{>0}{g'(\theta_r)}}{[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)]^2}\right\}<0$$

[18] Indeed, equation [8], like the standard *Beveridge Curve*, is a decreasing and convex function with respect to both v_r and v_s:

$$\frac{\partial^2 u}{\partial v_r^{\,2}}=\frac{-l\cdot\delta\cdot(\delta+\rho)^2\cdot \overset{<0}{g''(\theta_r)}\cdot H^2-\left\{-l\cdot\delta\cdot(\delta+\rho)^2\cdot g'(\theta_r)\cdot 2\cdot H\cdot(\delta+\rho)\cdot g'(\theta_r)\right\}}{H^4}>0$$

$$\frac{\partial^2 u}{\partial v_s^{\,2}}=\frac{-l\cdot\delta^2\cdot(\delta+\rho)\cdot \overset{<0}{g''(\theta_s)}\cdot H^2-\left\{-l\cdot\delta^2\cdot(\delta+\rho)\cdot g'(\theta_s)\cdot 2\cdot H\cdot\delta\cdot g'(\theta_s)\right\}}{H^4}>0$$

where $H\equiv[(\delta+\rho)\cdot g(\theta_r)+\delta\cdot g(\theta_s)+\delta\cdot(\delta+\rho)]$.

$$\frac{\partial u}{\partial v_s} = -\left\{ \frac{l \cdot \delta^2 \cdot (\delta + \rho) \cdot \overset{>0}{g'(\theta_s)}}{[(\delta + \rho) \cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta + \rho)]^2} \right\} < 0$$

Assuming as in Boeri and Garibaldi's (2006) calibrations that $\theta_r > \theta_s$, i.e. $v_r > v_s$ (which is a realistic situation also in the developing and transition countries), and knowing that $g'(\theta_i) > 0$, $g''(\theta_i) < 0$, we obtain $g'(\theta_s) > g'(\theta_r)$. Hence, if there is no monitoring ($\rho = 0$), the unemployment rate increases when the irregular vacancies decreases, because the *Beveridge Curve* of the hidden sector is steeper than the *Beveridge Curve* of the official sector, i.e. $\partial u / \partial v_s > \partial u / \partial v_r$.

However, a positive level of monitoring is a necessary condition to preserve legal jobs. Indeed, there is an efficient level of monitoring which reverse the previous result:

$$\rho > \{\delta \cdot [g'(\theta_s) / g'(\theta_r) - 1]\} \equiv \sigma$$

which is a positive value since $[g'(\theta_s) / g'(\theta_r)] > 1$. If $\rho > \sigma$, then the unemployment rate increases when the irregular vacancies increases, because now is the *Beveridge Curve* of the official sector to be steeper.

Note that in the inverse case ($\rho < \sigma$) we cannot ensure that the monitoring rate is positive, since σ may be a very small value.

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Figures

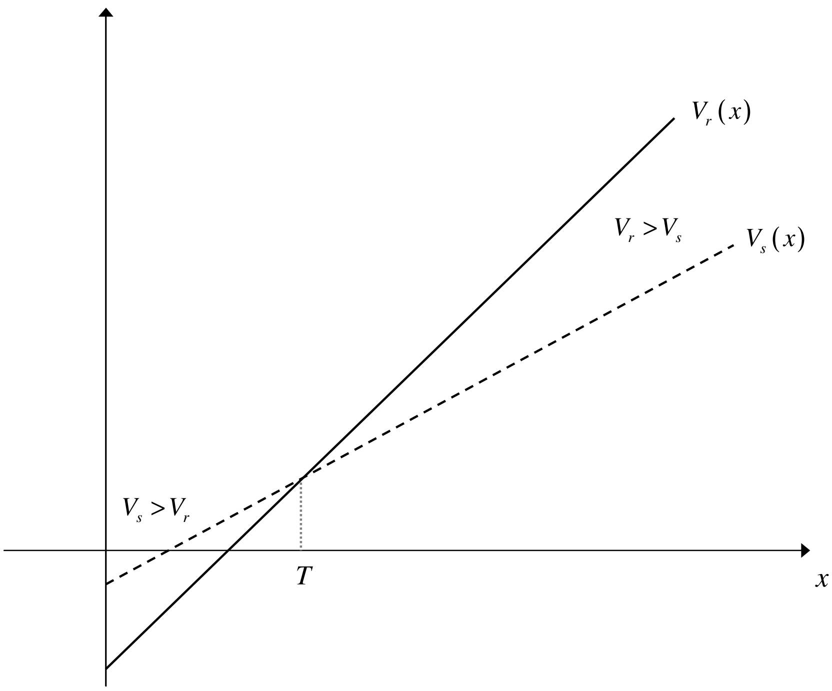


Figure 1. Entrepreneur' indifference condition

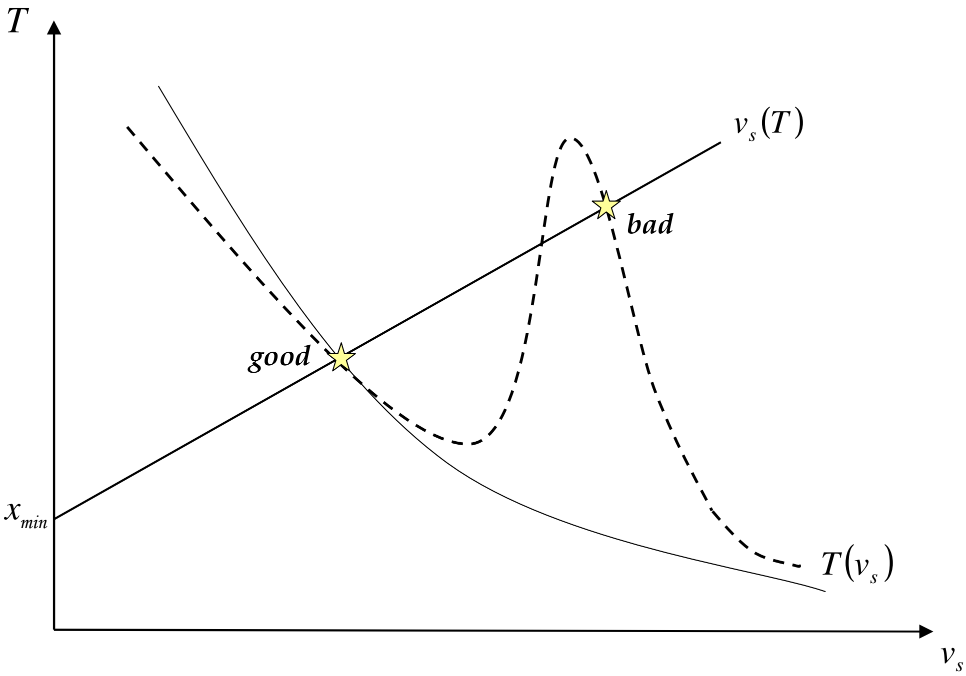
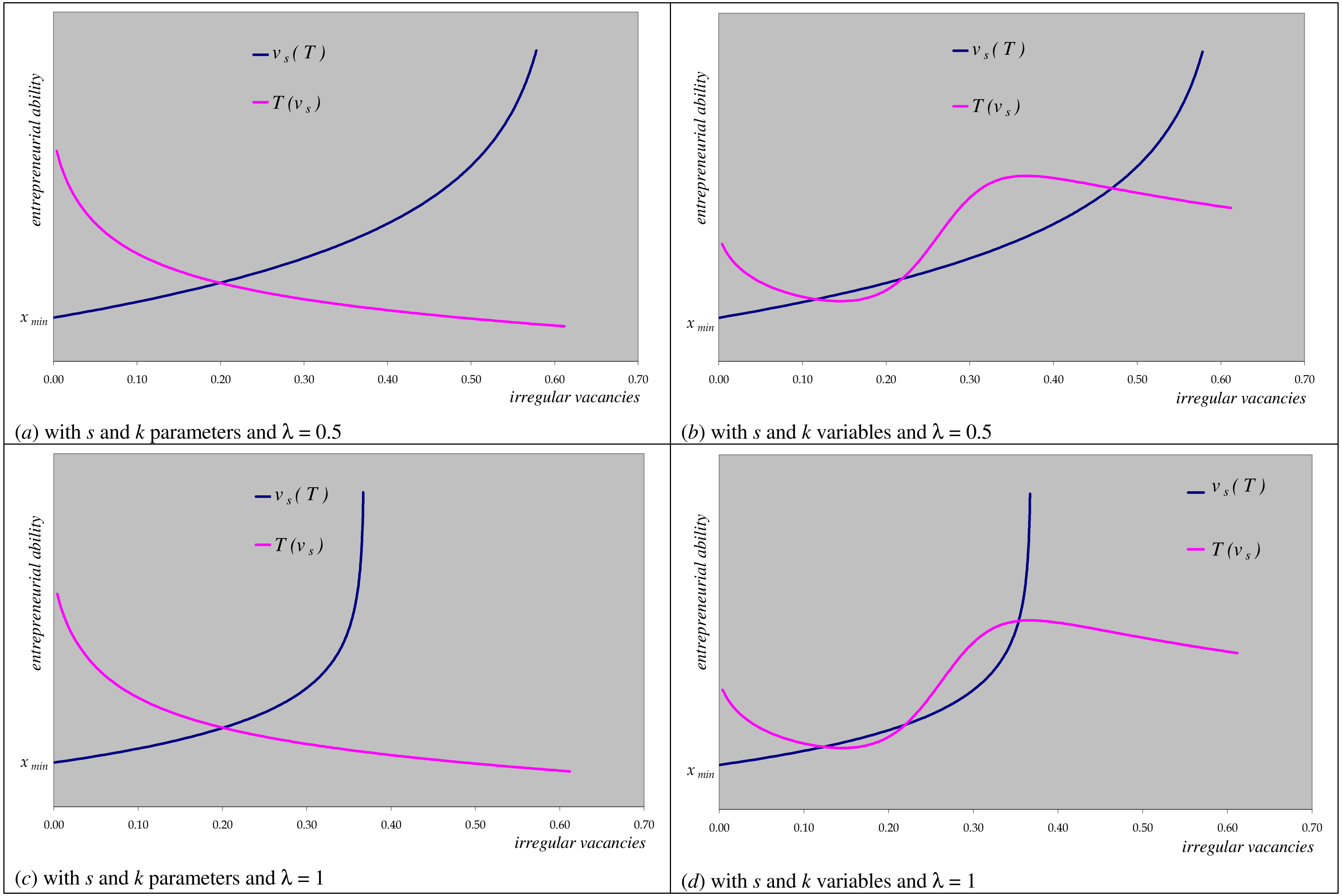


Figure 2. Interior equilibrium and multiple equilibria



(a) with s and k parameters and $\lambda = 0.5$

(b) with s and k variables and $\lambda = 0.5$

(c) with s and k parameters and $\lambda = 1$

(d) with s and k variables and $\lambda = 1$

Figure 3. General Equilibrium with Exponential Distribution
Source: Authors' calculations